A New Keynesian Model with Staggered Price and Wage Setting under Learning

Gabriela Best[1]

Abstract

This paper provides a study of the implications for economic dynamics when the central bank sets its nominal interest rate target in response to variations in wage inflation. I provide results on the existence, uniqueness, and stability under learning of rational expectations equilibrium for alternative specifications of the manner in which monetary policy responds to economic shocks when nominal rigidities are present. Monopolistically competitive producers set prices via staggered price contracts, and households set nominal wages in the same fashion. In this setting, the conditions for determinacy and learnability of rational expectations equilibrium differ from a model where only prices are sticky. I find that when the central bank responds to wage and price inflation and to the output gap, a Taylor principle for wage and price inflation arises that is related to stability under learning dynamics. In other words, a moderate reaction of the interest rate to wage inflation helps to avoid instability under learning and indeterminacy.

Keywords:
Learning, Monetary Policy, Nominal Wage and Price Rigidity, Expectational stability. *JEL classification: E52, E31, E43, E58.*

1. Introduction

The New Keynesian model has become a workhorse for the study of monetary policy in recent years. In this model, the behavior of private agents depends not only on current policy but also on the expected course of monetary policy. Monetary models typically assume that authorities adopt either linear feedback monetary rules (Taylor-type rules) or optimal monetary policy rules in an attempt to control

Email address: `gbest@fullerton.edu` (Gabriela Best)
[1]California State University, Fullerton, Department of Economics, Steven G. Mihaylo College of Business and Economics, Fullerton, CA 92834-6848. Telephone: (657) 278-2387; fax: (657) 278-3097.

the economy. However, the role of said rules in stabilizing the economy has been criticized because of their potential to induce indeterminacy or very large sets of rational expectations equilibria. If the central bank follows a rule that leads to multiple equilibria, agents might be incapable of coordinating on a specific equilibrium; even when capable of such coordination, this equilibrium might not be targeted by the central bank due to its undesirable characteristics.[2]

Alternatively, under certain conditions, agents can "learn" the desirable equilibrium targeted by the central bank and eventually converge to the rational expectations equilibrium (REE). Agents "learn" the equilibrium of the model by making forecasts based on recursive least squares techniques and the data obtained from the economy. These forecasts replace rational expectations (RE) in the model. In this context, the expectational stability (E-stability) concept discussed in Evans and Honkapohja (1999) and (2001) is applied in order to determine whether rational expectations equilibria are stable under learning dynamics.[3] When equilibria are E-stable, even when there are discrepancies between the agents' expectations and expectations required to yield a determinate REE, the system will converge to the REE. Therefore, designing rules that lead to learnable equilibria is desirable. Evans and McGough (2005a) studied determinacy and learnability conditions as selection criteria. Bullard and Mitra (2002, BM hereafter), derived determinacy and learnability conditions for monetary policy linear feedback rules, and Evans and Honkapohja (2003) determine the same conditions for optimal rules.[4] Others have examined them for open-economy models.[5]

Recent work has shown that staggering of nominal wage contracts is important to give rise to the key frictions that render monetary policy non-neutral. In fact, Christiano et al. (2005) conclude that wage stickiness—not price stickiness—appears more important in explaining output and inflation dynamics. Models that consider only sticky prices and not sticky wages have been criticized for producing "too sharp a real-wage decline in response to a tightening of monetary policy" as

[2]The large multiplicity of solutions and its harmful implications including equilibrium responses to shocks to fundamentals and sunspot states that could lead to arbitrarily large fluctuations in endogenous variables, have been widely discussed in Bullard and Mitra (2002), and Woodford (1999) and (2003).

[3]"Learnability," "E-stability," "stability under learning" are synonyms in the current text.

[4]They propose that central banks should adopt an optimal policy rule that includes both expectations and fundamentals to ensure determinacy and learnability of the REE.

[5]Authors include Llosa and Tuesta (2008), Bullard and Schaling (2009), Bullard and Singh (2008), Zanna (2009), and Wang (2006). These authors examine rules that respond to exchange rate movements. Moreover, extensions of the model that consider determinacy and Expectational stability of REE when long-term interest rates are included in the model are studied in McGough et al. (2005), and Kurozumi and Van Zandweghe (2008). Duffy and Xiao (2011) and Pfajfar and Santoro (2012) examine these rules in the context of models featuring physical capital. Evans and Honkapohja (2009) provide a comprehensive overview of recent literature on expectations, learning, and monetary policy.

addressed in Christiano et al. (1999). Christiano et al. (2005), Altig et al. (2011), and Smets and Wouters (2007) further conclude that impulse response functions after a monetary policy shock are best fit by the model with staggered wage contracts. This explanation validates wage stickiness as an important factor in explaining the real effects of monetary policy.

BM evaluate alternative monetary policy rules in the context of determinacy and E-stability in a standard New Keynesian model under price rigidity but wage flexibility. The authors conclude that the equilibria can be learnable when the central bank raises its interest rate instrument more than one-for-one with increases in inflation. This condition is referred to the "Taylor principle condition." It is not obvious that an optimizing-agent model with staggered nominal wage setting in addition to staggered price setting would yield determinacy and expectational stability (E-stability) conditions similar to a model in which only prices are sticky. One reason is that the volatility of aggregate wage inflation induces inefficiencies in the distribution of employment across households (Erceg et al., 2000). For that reason, this paper builds on BM and develops a comprehensive and systematic study of the determinacy and E-stability properties of the New Keynesian model under both price and wage rigidity. In particular, its contribution is to evaluate the E-stability properties of different monetary policy rules that embed an explicit response to wage inflation. Previous studies had concentrated only on documenting the **determinacy** properties of the staggered prices and wages model (see, e.g., Flaschel et al., 2008; and Franke and Flaschel, 2009; Galí, 2008). Galí (2008), through a numerical experiment, finds that if the interest rate reacts more than one to one to contemporaneous price inflation or wage inflation, then the REE is determinate. Flaschel et al. (2008) and Franke and Flaschel et al. (2009) confirm this result analytically by reformulating the model in continuous time.

This paper also relates to Huang et al. (2009), Ascari et al. (2011), and Carlstrom and Fuerst (2007), who study wage rigidities as a "special case" of their models. Huang et al. (2009) initially present a sticky price model with endogenous investment and find that incorporating both sticky wages and firm-specific capital makes the determinacy region quite large. Carlstrom and Fuerst (2007) analyze whether monetary policy should respond to asset prices in a model with price and wage stickiness from the point of view of equilibrium determinacy. They conclude that equilibra are likely to be indeterminate when the central bank adjusts policy in response to asset price movements. Lastly, Ascari et al. (2011) present an estimated monetary policy rule that includes a time-varying trend inflation and stochastic coefficients in a New Keynesian model for the U.S. economy and study its determinacy properties. Results suggest that including wage stickiness makes the determinacy region very sensitive to trend inflation. However, none of these papers focus on E-stability in the presence of wage stickiness.

The analysis presented in this paper views the short-term interest rate as the instrument of monetary policy design. The policy-design problem lies in characterizing how the interest rate should respond to changes in wage and price inflation to induce a learnable equilibrium given that both prices and wages exhibit rigidities. Wage inflation provides information about the rate of core inflation (De Long, 1997). Having a central bank that targets wage inflation in its policy rule is desirable because such rules (i) are welfare enhancing, performing nearly as well as the optimal rule, (Casares, 2007; Canzoneri et al., 2005; Erceg et al., 2000; Levin et al., 2006; Marzo, 2009)[6]; (ii) are simple (Levin et al., 2006) and have a good empirical fit to the data (Casares, 2007); and (iii) maximize economic stability (Mankiw and Reis, 2003). The underlying reason why wage inflation targeting is so desirable in the presence of price and wage rigidities is that wage rigidities create cross-sectional wage dispersion across households, which leads to inefficiencies in hiring decisions. In this setting, the cost of aggregate employment volatility is amplified. Stabilizing wage inflation eases wage dispersion and decreases the inefficiencies in employment.

The main results of this paper are twofold. First, a Taylor principle condition for wage and price inflation emerges when the central bank responds to wage inflation, price inflation, and the output gap. When the central bank adjusts its interest rates positively and more than one for one with changes in price and/or wage inflation above target, a "leaning against the wind" policy is followed. If agents do not have RE and they form forecasts using least squares learning, then such policy from the central bank pushes the equilibrium toward the REE. Thus, a leaning against the wind policy for a combination of wage and price inflation when agents form forecasts using least squares learning is closely linked to learnable equilibria. This result holds when interest rates respond to current data and forward-looking expectations. Second, there are instances in which having a central bank that responds mainly to wage inflation is preferable to responding to price inflation in its policy rule. Specifically, if the relative role of wage stickiness is more important in practice than the role of price stickiness, responding primarily to wage inflation in the policy rule results in a larger determinacy and E-stability regions of the parameter space. Responding to wage inflation, in this setting, relaxes the upper bound constraint on the response to wage inflation that appears in forward looking and lagged rules which ensures determinacy and E-stability.

From a welfare theoretic perspective, Erceg et al. show that the welfare cost of wage inflation volatility increases with the mean duration of wage contracts. They advocate implementing mixed rules that respond to wage and price inflation. To conclude, the result presented here supports Erceg et al. (2000) and Mankiw and Reis (2003) in the sense that it is desirable to design rules that respond to a combination of price and wage inflation due to their potential to induce determinacy and E-stability.

The rest of this paper is structured as follows. Section 2 presents the model, the alternative policy rules studied under the analysis, and the general conditions for E-stability akin to the model. Section 3 presents results on determinacy and learnability of equilibrium under alternative policy rules for a model with wage and price stickiness. Section 4 concludes.

[6]When the policymaker strictly targets price inflation in a model that includes staggered wage setting, there is a considerably large welfare loss due to substantial variation in the nominal wage inflation and the output gap.

2. The Environment

2.1. The Model

The structural equations of the supply side of the model are from Woodford (2003) (chapter 8, section 2.2), as follows:

$$\pi_t = \beta \, \widehat{E}_t \pi_{t+1} + \kappa_p (x_t + u_t) + \xi_p (w_t - w_t^n), \tag{1}$$

$$\pi_t^w = \beta \, \widehat{E}_t \pi_{t+1}^w + \kappa_w (x_t + u_t) + \xi_w (w_t^n - w_t), \tag{2}$$

$$w_t = w_{t-1} + \pi_t^w - \pi_t, \tag{3}$$

where $\kappa_p \equiv \xi_p \omega_p$ and $\kappa_w = \xi_w (\omega_w + \sigma^{-1})$, where $\xi_p = \frac{(1-\alpha_p)(1-\alpha_p \beta)}{\alpha_p (1+\omega_p \theta_p)}$ and $\xi_w = \frac{(1-\alpha_w)(1-\alpha_w \beta)}{\alpha_w (1+v\theta_w)}$.

Here π_t^w is nominal wage inflation, w_t is the log real wage, w_t^n represents exogenous variation in the natural real wage, x_t is the output gap, u_t is treated as an exogenous i.i.d. shock with variance σ_u^2, and $\widehat{E}$ represents (possibly nonrational) expectations. The terms ξ_p, ξ_w, κ_p, and κ_w are all positive. Prices and wages are adjusted à la Calvo, where $1-\alpha_p$ $(1-\alpha_w)$ is the time-independent probability that each of the prices (wages) is adjusted each period. The parameter ξ_p represents the sensitivity of goods-price inflation to changes in the average gap between marginal cost and current prices; it is smaller as prices are stickier (α_p). The parameter ξ_w indicates the sensitivity of wage inflation to changes in the average gap between households' "supply wage" (the marginal rate of substitution between labor supply and consumption) and current wages, and it is a function of the Calvo parameter that denotes wage stickiness in the economy (α_w). $\omega_p > 0$ represents the elasticity of supply wage with respect to the quantity supplied at a given wage, while $\omega_w > 0$ measures the elasticity of the supply wage with respect to the quantity produced; holding fixed households' marginal utility of income, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution. Eqs. (1) and (2) are Phillips curves for prices and wages. Eq. (3) is an identity for the real wage ($w_t = W_t/P_t$) expressed in logs and was rearranged in this form to provide a law of motion for the log of nominal wages.

The dynamic IS-type equation is described by

$$x_t = \widehat{E}_t x_{t+1} - \sigma (i_t - \widehat{E}_t \pi_{t+1} - r_t^n), \tag{4}$$

where i_t is the nominal interest rate and r_t^n is an exogenous i.i.d. shock with variance $\sigma_{r^n}^2$.[7] Monetary policy is represented by a Taylor-type rule that responds to price

[7] The exogenous shock r_t^n has been defined as an exogenous stochastic term that follows an AR(1) process in previous literature. This specification could potentially impact the E-stability conditions of the model. I abstain from this representation to avoid further complications in the derivation of the E-stability conditions of the model.

inflation, wage inflation, and the output gap. The monetary policy parameters are denoted by ψ_{π^w}, ψ_π, and ψ_x. The baseline specification is

$$i_t = \psi_\pi \pi_t + \psi_{\pi^w} \pi_t^w + \psi_x x_t. \tag{5}$$

This will be called the *contemporaneous data specification* because policymakers respond to contemporaneous data in their policy rules, and only the private sector forms expectations about future values of endogenous variables. The model consists of Eqs. (1) $-$ (5), they represent log-linear approximations of the equilibrium conditions outlined in Woodford (2003) and characterize only equilibria involving fluctuations around a zero inflation steady state.

2.2. Alternative Policy Rule Specifications

In addition to the baseline specification, I explore the determinate and E-stable regions of the parameter space when policymakers respond to possibly nonrational expectations of the policy variables, and lagged data in the policy rule with and without policy inertia. The first two specifications pertain to the cases without policy inertia:

This specification includes a policy function where the interest rate responds to current forecasts of one-quarter-ahead output gap, price inflation rate, and wage inflation rate. Forecast based rules describe well the conduction of monetary policy for the United States after 1979 as described in Best and Kapinos (2015) and Gali and Gertler (1998). This forward-looking policy function is represented by

$$i_t = \psi_\pi \widehat{E}_t \pi_{t+1} + \psi_{\pi^w} \widehat{E}_t \pi_{t+1}^w + \psi_x \widehat{E}_t x_{t+1} + \psi_i i_{t-1}. \tag{6}$$

where the term $\psi_i = 0$. The model now consists of Eqs. (1)-(4), and Eq. (5) is replaced by Eq. (6).

Alternatively, the central bank is more likely to respond to past data of the variables included in the policy function because contemporaneous data from the quarter in which they need to make policy decisions are rarely available. Specifically, the central bank has readily available data from the past quarter to which the interest rate should respond. For that reason, I include a policy function that responds to last-quarter data of the output gap, price inflation, and wage inflation. Equation (5) is now replaced by

$$i_t = \psi_\pi \pi_{t-1} + \psi_{\pi^w} \pi_{t-1}^w + \psi_x x_{t-1} + \psi_i i_{t-1}. \tag{7}$$

where ψ_i has also been set to 0.

As has been widely addressed in the literature (Bullard and Mitra, 2007; Dennis, 2006; Evans and McGough, 2005c; Cukierman, 1989; and Brainard, 1967), rules that respond cautiously to inadvertent changes in economic conditions are desirable. This caution can be modeled by having a central bank that responds to inertia on its policy rule. Equation (6) presents a forward-looking policy function with policy inertia in which the interest rate instrument responds to changes in the forecasts of price and wage inflation, and the output gap, as well as the lagged interest rate. Alternatively, Eq. (7) represents a policy rule in which the central bank responds not only to lagged variables, but also to a lagged interest rate term.

2.3. Determinacy

RE are viewed as a two-sided equilibrium in which expectations influence the time path of the economy and the time path of the economy affects expectations. A model is said to be determinate if it has a unique dynamically stable REE. The general conditions for determinacy are outlined below. Consider a general class of models:

$$y_t = \alpha + BE_t y_{t+1} + \delta y_{t-1} + \kappa e_t \tag{8}$$

where y_t is an $n \times 1$ vector of endogenous variables; e_t is a vector of white noise; and B, δ, and κ are $n \times n$ matrices of coefficients.

For determinacy analysis the δ matrix in (8) has been set to zero. In order to yield determinacy, the number of free variables in the model needs to be equal to the number of eigenvalues of matrix B with absolute value less than 1. Otherwise, the equilibrium is indeterminate.

When the model yields indeterminacy, there are multiple possible responses of the endogenous variables to shocks to fundamentals, some of which can create amplified economic fluctuations. However, endogenous variables can also respond to "sunspots" or extraneous random variables with no fundamental significance. Previous literature studies such as Evans and Honkapohja (2001) and Evans and McGough (2005b) discuss the existence of models in which the solutions depend on sunspots.[8]

2.4. Learning Methodology

The general conditions for E-stability are outlined below. Following the literature on learning in macroeconomics (e.g., Evans and Honkapohja, 2001) and considering a general class of models represented by Eq. (8) the MSV solutions take the form

$$y_t = a + by_{t-1} + ce_t, \tag{9}$$

with corresponding expectations

$$E_t y_{t+1} = (I + b)a + b^2 y_{t-1} + bce_t. \tag{10}$$

Inserting equation (10) into equation (8), it follows that the MSV solutions satisfy

$$(I - Bb - B)a = \alpha, \tag{11}$$

$$Bb^2 - b + \delta = 0, \tag{12}$$

$$(I - Bb)c = \kappa. \tag{13}$$

[8] In "regular" cases, these solutions are explosive, but in "irregular" cases, they are stationary. A regular linear model assumes that there exists a unique stationary REE. By contrast, in an irregular linear model, multiple stationary solutions are possible, particularly solutions that depend on sunspots. Changes in the variable (sunspot) could trigger self-fulfilling shifts in expectations and in the fundamentals in the model, creating disproportionately large fluctuations in the economy.

The actual law of motion (ALM) takes the form

$$y_t = \alpha + B(I+b)a + (Bb^2+\delta)y_{t-1} + (Bbc+\kappa)e_t. \tag{14}$$

To determine E-stability, I consider equation (9) as the PLM and the mapping from PLM to ALM takes the form

$$T(a,b,c) = (\alpha + B(I+b)a, Bb^2+\delta, Bbc+\kappa). \tag{15}$$

The expectational stability is determined by the following matrix differential equation:

$$\frac{d}{d\tau}(a,b,c) = T(a,b,c) - (a,b,c). \tag{16}$$

In order to analyze the local stability of system (16) at a RE solution $\overline{a}$, $\overline{b}$ $\overline{c}$, the system is linearized at that RE solution. The E-stability conditions are governed by the equation for a, b, c in (14). Using the rules for vectorization of matrix products, I compute

$$DT_a(\overline{a},\overline{b}) = B(I+\overline{b}), \tag{17}$$

$$DT_b(\overline{b}) = \overline{b}\prime \otimes B + I \otimes B\overline{b}, \tag{18}$$

$$DT_c(\overline{b},\overline{c}) = I \otimes B\overline{b}. \tag{19}$$

A particular MSV solution $(\overline{a},\overline{b},\overline{c})$ is E-stable if the MSV fixed point of the differential equation (15) is locally asymptotically stable at that point. Proposition 10.3 in Evans and Honkapohja (2001) states the conditions for E-stability of the MSV solution.

2.5. Parameters

In most cases analytic results are not tractable and so I proceed numerically as in Galí (2008), Evans and McGough (2007), and Bullard and Mitra (2007). The model was calibrated with parameter values from Amato and Laubach (2004).[9] They estimated impulse responses of wages and prices to a monetary policy shock. These parameters are considered the baseline calibration. The robustness of results are also verified under the alternative calibration from Giannoni and Woodford (2003).

In order to illustrates the learnable and/or determinate regions of the parameter space for the Taylor rule specifications previously outlined, Figures 1–5 are drawn in (ψ_x, ψ_π) space with all the other parameters set to their baseline values except for $\psi_{\pi^w} = 0$, 0.5, 1, and 1.5 and $\psi_i = 0$, 0.65, and 5. The wage inflation parameter value of 0 corresponds to a rule where the interest rate is responding only to price inflation and the output gap—the BM result. The values 0.5 and 1 correspond to a rule with a moderate to aggressive response to wage inflation in addition to the

[9] The authors extend the analysis of Rotemberg and Woodford (1997) by adding a real wage series to a VAR.

Parameters	Amato and Laubach (2004)
β	0.99
ω_p	0.33
ω_w	0.27
ξ_p	0.058
ξ_w	0.066
σ	1/0.26
ψ_π	$0, 0.5, 1, 1.5$
ψ_{π^w}	$0, 0.5, 1, 1.5$

Table 1: Parameter Values Baseline Calibration

response to the output gap and price inflation. In various estimations of a Taylor rule for the United States, where the short-term interest rate responds to price inflation and the output gap, the coefficient for price inflation was 1.5 and for the output gap was 0.5. These parameter values were found to characterize U.S. policy between 1987 and 1992 as addressed in Woodford (2003). The parameter value of 1.5 was the largest value chosen because it is within the range of reasonable policy parameter values, although the price inflation policy coefficient has often been estimated to be greater than 1.5.

Additionally, I discuss the learnable and/or determinate regions in the (ψ_x, ψ_{π^w}) space for $\psi_\pi = 0$ when responding uniquely to wage inflation and the output gap results in different determinate/E-stable regions from those obtained under a purely price inflation and output gap stabilization goal. This perspective allows me to perform a sensitivity analysis using two-dimensional snapshots of regions in the three-dimensional space (ψ_x, ψ_π, and ψ_{π^w}).

3. Policy Rules under Determinate and Learnable Equilibria

3.1. Contemporaneous Data in the Policy Rule

3.1.1. Determinacy

The model can be simplified by substituting the policy rule (5) into (4) and writing the system involving the endogenous variables x_t, π_t, π_t^w, and w_{t-1} given by Eqs. (1), (2), (3), and (4) in the form of Eq. (8) (with δ set to zero), reproduced here for convenience:

$$y_t = \alpha + BE_t y_{t+1} + \kappa e_t \tag{20}$$

where y_t=[x_t, π_t, π_t^w, w_{t-1}]$'$, $\alpha = w_t^n$, e_t=[${r_t}^n$, u_t]$'$, and matrix B is defined as in Appendix A.

In this setting, x_t, π_t, and π_t^w are free variables. For that reason, three of the four eigenvalues of the system need to be inside the unit circle for determinacy; otherwise, the equilibrium is indeterminate. Of note, the parameter space consistent with determinate equilibrium is identical to the one with E-stability.

3.1.2. Learning

I assume that y_t is not available when the forecasts $\widehat{E}_t y_{t+1}$ are formed, and the information set is represented by $(1, y_{t-1}, e_t')$.[10] The model is written as

$$\widehat{y}_t = \alpha + \widehat{B} E_t \widehat{y}_{t+1} + \widehat{\delta} \widehat{y}_{t-1} + \widehat{\kappa}\ e_t, \tag{21}$$

where $\widehat{y}_t$=$[x_t,\ \pi_t,\ \pi_t^w,\ w_t]\prime$, $\alpha = w_t^n$, e_t=$[r_t{}^n,\ u_t]\prime$, and matrices $\widehat{B}$, $\widehat{\delta}$, and $\widehat{\kappa}$ are defined as in Appendix A.

The MSV solution takes the form of Eq. (9).[11] Equation (9) for $\bar{b}$ is characterized by a matrix quadratic that could have multiple solutions. The determinate equilibrium corresponds to the case where there is a unique solution for $\bar{b}$ with all of its eigenvalues inside the unit circle. This paper analyzes stability under adaptive learning of REE that are asymptotically stationary. It is possible to analyze stability under adaptive learning of explosive solutions but here I focus on the solutions that are asymptotically stable. The solution to matrix $\bar{b}$ was obtained using the "trust-region-dogleg" method for systems of non-linear equations (for a detailed description see Powell, 1970).

E-stability of the MSV solution under learning is now considered. The PLM takes the form of the MSV solution. The mapping from PLM to ALM takes the form of Eq. (15). To compute the E-stability conditions I use derivatives (17), (18), and (19), where the three matrices require real parts less than 1 for E-stability. If at least one of the eigenvalues of the matrices has a real part greater than 1, then the equilibrium is E-unstable. After finding the MSV solution, E-stability conditions were numerically evaluated. The results are discussed in the next subsections.

3.1.3. Determinacy and E-stability

Figure 1 plots the region of determinacy and E-stability of the MSV solution as a function of ψ_π and ψ_x with all the other parameter values set at baseline values, and where ψ_{π^w} takes values of 0, 0.5, 1 and, 1.5. The top left panel shows the determinacy area for a contemporaneous data rule that responds only to output gap and price inflation , corresponding to the BM result. A negatively sloped line consistent with the Taylor principle emerges–modified Taylor principle boundary hereafter. Points to the right of the line correspond to a determinate and E-stable REE, and points to the left are indeterminate and E-unstable. The line has a vertical intercept of $\psi_x = \frac{\kappa}{1-\beta} = 2.65$, where $\kappa = \frac{\kappa_w \xi_p + \kappa_p \xi_w}{\xi_p + \xi_w}$ and a horizontal intercept of $\psi_\pi = 1$.

As the response to wage inflation increases, the modified Taylor principle boundary shifts toward the origin. Thus, when the response to wage inflation takes the

[10] The timing convention of the information set $(1,\ y_{t-1},\ e_t\prime)$ is standard in the literature, as suggested by Evans and Honkapohja (2009) review paper. It avoids simultaneity between expectations and outcomes.

[11] Notice that only the lagged value of w_t enters Eq. (9) under a contemporaneous data specification of the Taylor rule.

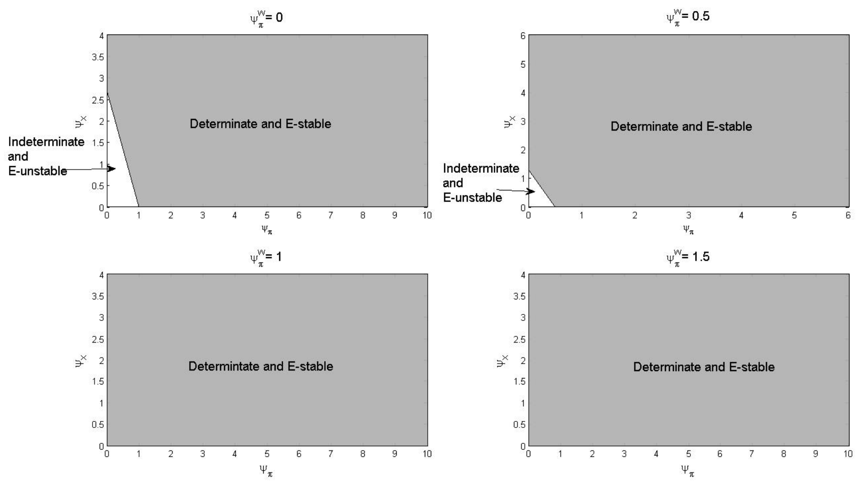

Figure 1: Determinacy and learnability for a model with contemporaneous data in the policy rule. All parameters except for ψ_π and ψ_x are set at baseline values.

value of ($\psi_{\pi^w} = 0.50$), the response to price inflation must also be 0.50 in order to induce determinacy and E-stability. In fact, determinacy and E-stability under a contemporaneous data rule are governed by the following modified version of the Taylor principle for a combination of wages and prices:

$$\psi_\pi + \psi_{\pi^w} + \frac{(1-\beta)}{\kappa}\psi_x > 1. \tag{22}$$

Eq. (22) is consistent with the numerical analysis and its derivation is discussed in Appendix A. This reformulated Taylor principle matches the determinacy condition of Flaschel et al. (2008) and Flaschel and Franke (2009).[12]

In order to shed some light on the economic interpretation of this reformulated Taylor principle, Eqs. (1) and (2) can be redefined using a particular weighted average of wage and price inflation $\tilde{\pi} = \frac{\xi_p^{-1}\pi_t + \xi_w^{-1}\pi_t^w}{\xi_p^{-1}+\xi_w^{-1}}$, as in Woodford (2003). Now the Phillips curves reduce to:

$$\tilde{\pi} = \bar{\kappa}(\hat{Y}_t - \hat{Y}_t^n) + \beta E_t \tilde{\pi}_{t+1}. \tag{23}$$

The coefficient $\bar{\kappa} = \frac{\sigma^{-1}+\omega}{\xi_p^{-1}+\xi_w^{-1}}$, which is identical to κ, becomes smaller the greater

[12] I drew values of ψ_π, ψ_{π^w}, and ψ_x from a uniform distribution [0,10]. I recorded whether the equilibrium was determinate and E-stable for values of $\psi_\pi + \psi_{\pi^w} > 1$. I repeated this exercise 1 million times; in 100 percent of the cases when $\psi_\pi + \psi_{\pi^w} > 1$, the equilibrium was determinate and E-stable.

the degree of rigidity of either wages or prices. When only wages (prices) are sticky, Eq. (23) becomes a Phillips curve for wages (prices). Using the methodology in Ascari and Ropele (2009), from Eq. (23) it can be observed that each percentage point of permanently higher weighted average of inflation leads to a long-run increase in the output gap of $((1-\beta)/\kappa)$ percentage points. Thus the left hand side of Eq. (22) represents the long run increase in the nominal interest rates proposed by the policy rule (5) for each unit of permanent increase in the inflation rate. In any case, the Phillips curve for wages is analogous to the Phillips curve for prices, therefore, when the Taylor principle is satisfied, a departure of the private sector expected inflation (price and/or wage) from RE value will increase the real interest rate. The increase in the real interest rate would reduce output through Eq. (4) leading to a reduction of inflation through Eqs. (1) and (2) or (23), under the traditional demand channel.[13]

Wage stickiness has proven to be a key element that improves the empirical fit of the model and explains the real effects of monetary policy. Incorporating wage stickiness could potentially alter equilibrium determinacy (Ascari et al., 2011; Carlstrom and Fuerst, 2007; and Huang et al., 2009) and E-stability conditions. I proceed by considering whether increasing the degree of wage stickiness, represented by decreasing ξ_w (going from 0.066, the value assigned in our baseline calibration, to 0.0042 in Giannoni and Woodford, 2003), affects equilibrium determinacy and E-stability. I find that a policy rule that responds to contemporaneous data has analogous implications for determinacy of REE and stability under learning, regardless of the source of rigidity. As κ decreases (due to stickier prices or wages), the modified Taylor principle boundary pivots downward, finding its vertical intercept at $\psi_x = \frac{\kappa}{1-\beta}$, expanding the determinate and E-stability region.

3.2. Forward Expectations in the Policy Rule

3.2.1. Determinacy

The model with forward-looking expectations is composed of Eqs. (1)-(4) and (6) for $\psi_i = 0$. The system can be reduced to four equations when I substitute Eq. (6) into Eq. (4). The analysis follows Section 3.1.1, with matrix B defined as in Appendix B.

Fig. 2 presents the numerical results. When the central bank responds only to forward expectations of price inflation and the output gap in the policy rule (Case 1) the determinate equilibrium occurs when $\psi_\pi > 1$, consistent with the modified

[13] This paper also analyzes determinacy and E-stability conditions for a Taylor rule that is responding to contemporaneous expectations in its policy feedback rule or nowcasting: $i_t = \psi_\pi \widehat{E}_t \pi_t + \psi_{\pi^w} \widehat{E}_t \pi_t^w + \psi_x \widehat{E}_t x_t$. Policymakers condition their policy instruments on expected values of current wage and price inflation, and the output gap. Determinacy and E-stability conditions are identical to the case with contemporaneous data in the policy feedback rule under the assumption present in this paper regarding observability of exogenous processes. I follow the conditions for determinacy and E-stability outlined in Evans and McGough (2005a).

Taylor principle condition, and $\psi_x < 0.52$. Under the benchmark calibration, as the response to price (wage) inflation increases, the response to the output gap should gradually decrease concurrently in order to yield a determinate REE–e.g., when $\psi_\pi = 15$ ($\psi_{\pi^w} = 15$), $\psi_x < 0.38$ ($\psi_x < 0.27$) ensures determinacy.

The numerical analysis matches the following boundary that divides the parameter space into determinate and indeterminate regions

$$\psi_x = \frac{a_1 + \sigma(\kappa_w \xi_p + \kappa_p \xi_w)(1 - \psi_\pi - \psi_{\pi^w}) - 2\sigma(\beta + 1)(\psi_\pi \kappa_p + \psi_{\pi^w} \kappa_w)}{(\beta + 1)(2 + 2\beta + \xi_p + \xi_w)\sigma} \tag{24}$$

where $a_1 = (2\xi_p + 2\xi_w + 2\kappa_p \sigma)(1 + \beta) + 4(\beta + 1)^2$.

This boundary is similar to Eq. (40) from BM and it has also been documented by Galí (2008) and Bernanke and Woodford (1997) for rules that respond only to expected future price inflation and expected future output gap under price stickiness.[14] It posits an upper bound constraint on response coefficients which shows up in forward-looking rules that guarantees equilibrium determinacy.

In the current analysis, determinacy of equilibrium under forward looking rules requires that (i) the central bank should respond neither too strongly nor too weakly to price and/or wage inflation, and not too strongly to the output gap. Therefore, the Taylor principle condition is insufficient to guarantee determinacy; adjusting interest rates strongly in response to changes in expected inflation or the expected output gap can lead to equilibrium fluctuations attributed to self-fulfilling expectations. (ii) The central bank should also be mindful of the degrees of price and wage stickiness prevailing in the economy when conducting its policy design. The determinacy area is now sensitive to the degree of price and wage rigidity through its effects on κ_w and κ_π, along with the type of inflation (π or π^w) that the central bank chooses to target.

If $\kappa_p = \kappa_w$, then determinacy would not be affected by whether the central bank decides to respond to expected future π or π^w in the Taylor rule. In practice these parameters are not necessarily equal. For example, the benchmark values used herein are $\kappa_p = 0.0191$ and $\kappa_w = 0.0350$. Even when the quantitative differences are not large for the calibration currently used, the results highlight the relative roles of price and wage stickiness for policy feedback rules that respond to expected future variables.[15] If one type of stickiness was more important in practice this would lead to even stronger implications for policy. When, for instance, wage stickiness is higher, so that $\kappa_w < \kappa_p$, a policy that responds to wage inflation has a larger determinacy region. Responding to wage inflation pivots upward the determinacy boundary (24) and it ensures that the equilibrium is determinate even for high values of ψ_{π^w}. Thus, having a central bank that tries to demonstrate the seriousness with which it takes its inflation target is not problematic because responding particularly strongly to a type of inflation forecast is not conducive to indeterminacy.

[14] Details on its derivation are included in Appendix B.

[15] This is also the case for lagged data rules, discussed later in the text.

Under Eq. (5), wage and price inflation have symmetric contemporaneous effects on the output gap through the interest rate. However, when the central bank responds to expected price and wage inflation as in Eq. (6), expected price inflation enters directly through the interest rate **and** through the IS equation, while expected wage inflation affects output only through the interest rate. Therefore, it is not surprising that the forward looking case postulates different underlying implications for determinacy from the contemporaneous data case.

3.2.2. E-stability

The E-stability conditions for rules with forward-looking expectations in the policy rule are akin to the analysis in Section 3.1.2. Matrices $\widehat{B}$, $\widehat{\delta}$, and $\widehat{\kappa}$ are defined in Appendix B.[16]

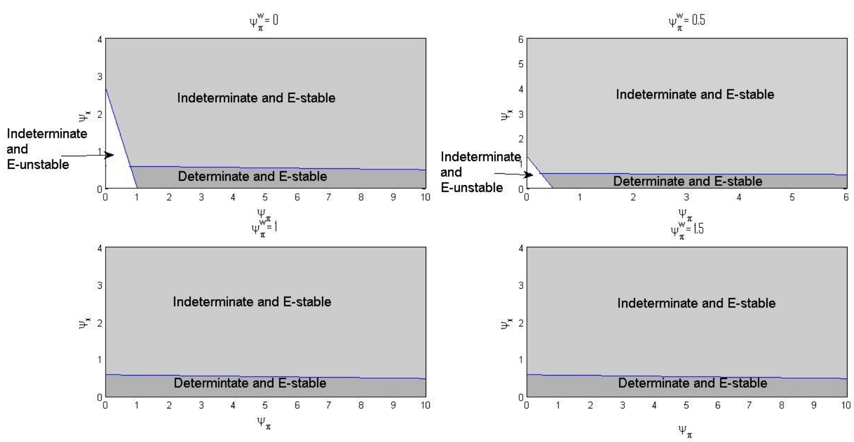

Figure 2: Determinacy and learnability for a model with forward expectations in the policy rule. All parameters except for ψ_π and ψ_x are set at baseline values.

Fig. 2 plots the E-stable region of the MSV solution. The modified Taylor principle given by Eq. (22) is sufficient to guarantee E-stability of REE. The MSV solution is learnable even when found in the indeterminate region of the parameter space; the converse, however, is not true.

3.3. Lagged Data in the Policy Rule

3.3.1. Determinacy

The model encompasses Eqs. (1)-(4) and (7), where Eq. (7) for $\psi_i = 0$ has been moved one period forward and is represented in Appendix C. The matrix of expectational variables pre-multiplies the inverse of the first (left hand) matrix from

[16] Only the lagged value of w_t enters Eq. (10) under the forward-looking policy function.

Eq. (C.1) resulting in Matrix B, which is the appropriate matrix for determinacy analysis. Determinacy analysis follows Section 3.1.1.

3.3.2. Determinacy

Fig. 3 plots the determinacy region. When the central bank responds to lagged price inflation and lagged output gap the modified Taylor principle boundary and the determinacy boundary (24) appear. Two determinacy regions emerge: Region 1 for values of $\psi_\pi > 1$ and a modest value for ψ_x (less than 0.52), analogous to the determinacy region under forward expectations in the policy rule, therefore sensitive to changes in wage stickiness.

Region 2 for values to the left of the negatively sloped line, where ψ_π is at most 0.8 and $0.52 < \psi_x < 2.65$. Determinacy in this setting posits a trade-off between (i) a relatively large response to lagged inflation (greater than 1) and a relatively small response to lagged output gap or (ii) a relatively large response to lagged output gap and a relatively small response to lagged price inflation. Therefore, with lagged data in the policy rule the Taylor principle condition is neither necessary nor sufficient to guarantee uniqueness of equilibrium. As the combined response to inflation increases and achieves a value greater than 1, the only determinate area that prevails is Region 1.

3.3.3. E-stability

The model was reshaped as in Section 3.1.2. See Appendix C for matrices $\widehat{B}$, $\widehat{\delta}$, and $\widehat{\kappa}$. E-stability analysis follows Section 3.1.2.

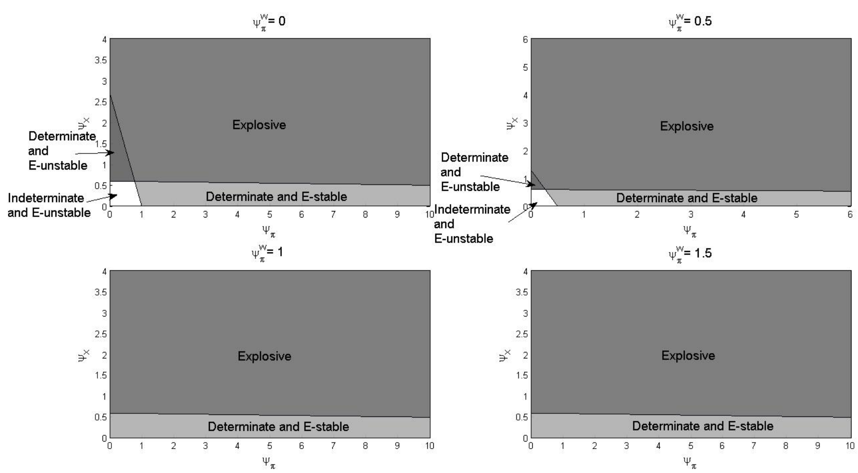

Figure 3: Determinacy and learnability for a model with lagged data in the policy rule. All parameters except for ψ_π and ψ_x are set at baseline values.

Fig. 3 shows the E-stable regions of the MSV solutions. The E-stable region of the parameter space is analogous to determinacy Region 1. The upper-left panel is qualitatively close to BM, but the position of the line separating the explosive region and the determinate and E-stable region is now affected by the degree of wage stickiness.

3.4. Policy Inertia

In the policy inertia case, the systems of equations are given by (1)-(4), and (6) for the system with forecast data, and Eq. (7) for the system with lagged data, and policy inertia in the monetary policy rule. The analysis of determinacy and E-stability of the MSV solution is standard and follows Sections 2.3 and 2.4.[17] As in Bullard and Mitra (2007), analytical solutions were not obtained for the E-stability conditions of stationary MSV solutions, however the results are discussed using the baseline calibration values of Amato and Laubach (2004).

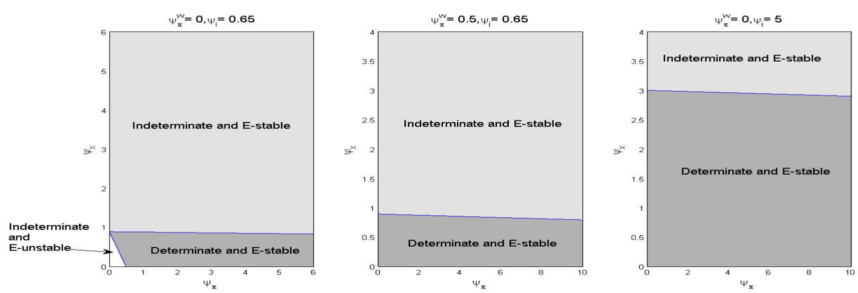

Figure 4: Determinacy and learnability for a model with policy inertia in the forward looking policy rule. Parameter ψ_i takes on values of 0.65, 0.65 and 5, and ψ_{π^w} values of 0, 0.5, and 0. All other parameters except for ψ_π and ψ_x are set at baseline values.

Under the policy rule that responds to forward expectations and policy inertia, the results, plotted in Fig. 4, compare quantitatively to the graphs without policy inertia, Fig. 3, as follows: when $\psi_i = 0.65$ without any response to wage inflation (first panel of Fig. 4 from left to right), the horizontal line that divides the parameter space shifts upward and now has an intercept of roughly 0.90 (before it was 0.52); the determinacy and learnable area is now more prominent. The second panel on Fig. 4 shows a further increase in the desirable area as the response to wage inflation increases ($\psi_{\pi^w} = 0.50$). Moreover, a strong response to policy inertia $\psi_i = 5$ yields a an even larger determinate and E-stable region (third panel of Fig. 4); however, the same desirable region prevails for higher values of ψ_{π^w} (i.e.

[17]The matrices obtained when performing the determinacy and E-stability of stationary MSV solution analyses are available upon request.

0.50, 1.00 and 1.50). It can also be noted that active Taylor-type rules may lead to E-stable indeterminacy under the present calibration.[18]

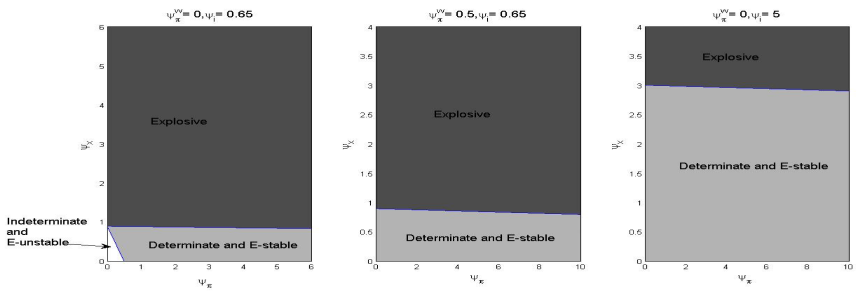

Figure 5: Determinacy and learnability for a model with policy inertia in the lagged policy rule. Parameter ψ_i takes on values of 0.65, 0.65, and 5, and ψ_{π^w} values of 0, 0.5 and 0. All other parameters except for ψ_π and ψ_x are set at baseline values.

When the policy rule responds to lagged data and policy inertia, results represented in Fig. 5, the determinate and E-stable areas are similar to the three panels of Fig. 4 with two important differences: An increase in the policy inertial parameter ψ_i to 0.65 eliminates the determinate and E-unstable area previously described as Region 2 in Section 3.3. Furthermore, the region above the desirable area is now explosive.[19] The prospect of rules that include policy inertia of yielding a determinate equilibrium that is learnable is enhanced as the policymaker adjusts its instrument in response to changes in—forecasts or lagged—price and wage inflation under sticky prices and wages.

4. Conclusion

The study of determinacy and stability under learning with various specifications of Taylor type-rules in a model with only price rigidities was developed by Bullard and Mitra (2002). I build on their work by considering the determinacy and learnability conditions in a model where monopolistically competitive firms and households set prices and wages in staggered contracts following Erceg et al. (2000). Furthermore, I consider alternative specifications of a nominal interest rate rule followed by the central bank that responds not only to price inflation and the

[18] For a further analysis on forward looking rules and stable indeterminacy with inertia refer to Evans and McGough (2005c).

[19] The previous exercise was also conducted using parameter values from Bullard and Mitra (2007) and we obtain analogous results; (i) Region 2 disappears and (ii) the horizontal line that divides the parameter space shifts upward, making the determinate and learnable region larger in the presence of sticky wages and prices.

output gap, but also to wage inflation. The main result is twofold. First, when the central bank responds to wage and price inflation and the output gap, a modified Taylor principle for wage and price inflation arises: The nominal interest rate should be adjusted more than one for one with changes in (wage and/or price) inflation. This Taylor principle is closely linked with stability under learning dynamics when the central bank responds to current data and forward-looking expectations. Furthermore, when the central bank adjusts the interest rate in response to lagged data, the policy instrument must (i) respond moderately to changes in the output gap and wage/price inflation and (ii) meet the modified Taylor principle condition in order to yield stability under learning dynamics.

Second, results show that sticky wages and sticky prices have an impact on the determinacy and E-stability areas of the parameter space. If one type of stickiness was more important in practice this would lead to strong implications for policy design. When, for instance, wage stickiness plays a more significant role that price stickiness, it is preferable to target wage inflation than price inflation because such policy yields a larger determinacy and E-stable region. In particular, it relaxes the upper bound constraint on the central bank's response to wage inflation necessary to ensure determinacy and E-stability. Thus, having a central bank that attempts to demonstrate the seriousness with which it takes its inflation target is no longer an issue, because responding extremely vigorously to wage inflation is not conducive to indeterminacy or instability under learning dynamics. This result supports Woodford (2003) and Erceg et al. (2000) in the sense that the degree of wage and/or price stickiness affects the monetary policy stabilization goals. They find that in the extreme case of only sticky wages, optimal policy entails complete stabilization of wages. In practice they advocate seeking to stabilize an appropriate weighted average of wage and price inflation. Herein, results suggest that a central banker concerned with avoiding indeterminacy and/or instability under learning should consider responding to wage inflation in addition to price inflation.

Acknowledgements: I especially thank Bill Branch for his advice and help in many aspects of this project. I also thank Michelle Garfinkel, Judy Ahlers, James Bullard, Pavel Kapinos, and anonymous referees for important input into this paper. I also want to thank the Department of Economics and School of Social Sciences at the University of California, Irvine for financial support.

5. References

[1] Amato, J., and Laubach, T. (2004). Implications of habit formation for optimal monetary policy.*Journal of Monetary Economics*, 51, 305-325.

[2] Ascari, G., and Ropele, T. (2009). Trend Inflation, Taylor Principle and Indeterminacy. Bank of Italy Working Paper No. 708.

[3] Ascari,G., Branzoli, N. and E. Castelnuovo (2011). Trend Inflation, Wage Indexation, and Determinacy in the U.S. mimeo. University of Pavia.

[4] Altig, D., Lawrence, C., Eichenbaum, M., and Linde, J. (2011). Firm-specific capital, nominal rigidities and the business cycle. *Review of Economic Dynamics*, 14(2), 225-247.

[5] Bernanke, B., and Woodford, M. (1997). Inflation Forecasts and Monetary Policy. *Journal of Money, Credit and Banking*, 29(4), 653-684.

[6] Best, G. and Kapinos, P. (2015). In What Sense Is Monetary Policy Forward-Looking? mimeo. California State University, Fullerton.

[7] Brainard, W. (1967). Uncertainty and the effectiveness of policy. *American Economic Review* 57, 411-425.

[8] Bullard, J., and Mitra, K. (2002). Learning about monetary policy rules. *Journal of Monetary Economics*, 49, 1105-1129.

[9] Bullard, J., and Mitra K. (2007). Determinacy, learnability, and monetary policy inertia. *Journal of Money, Credit and Banking*, 39, (5) 1177-1212.

[10] Bullard, J., and Schaling, E. (2009). Monetary policy, determinacy, and learnability in a two-block world economy. *Journal of Money, Credit and Banking*, 41(8), 1585-1612.

[11] Bullard, J., and Singh, A. (2008). Worldwide macroeconomic stability and monetary policy rules. *Journal of Monetary Economics*, 55(Supplement), S34-S47.

[12] Canzoneri, M., Cumby, R., and Diba, B. (2005). Price and Wage Inflation Targeting: Variations on a Theme by Erceg, Henderson and Levin. in Jon Faust, Athanasios Orphanides and David Reifschneider, eds., Models and Monetary Policy: Research in the Tradition of Dale Henderson, Richard Porter, and Peter Tinsley, Board of Governors of the Federal Reserve System: Washington DC, 2005.

[13] Carlstrom C., Fuerst T. and Ghironi, F. (2006). Does it matter (for equilibrium determinacy) what price index the central bank targets? *Journal of Economic Theory*, 128,1, 214-231.

[14] Carlstrom, C., and Fuerst T. (2007). Asset prices, nominal rigidities and monetary policy. *Review of Economic Dynamics*, 10, 256-275.

[15] Casares, M. (2007) Monetary policy rules in a New Keynesian euro area model. *Journal of Money, Credit and Banking*, 39, 4, 875-900.

[16] Christiano, L., Eichenbaum, M., and Evans, C. (1999). Monetary Policy Shocks: What Have We Learned and to What End? In *Handbook of Macroeconomics* 1A.

[17] Christiano, L., Eichenbaum, M., and Evans, C. (2005). Nominal Rigidities and the Dynamic effects of a Shock to Monetary Policy. *Journal of Political Economy* 113, 1-45.

[18] Clarida, R., Gali, J., and Gertler, M. (1998). Monetary Policy Rules in Practice: Some International Evidence. *European Economic Review* 42, 1033-1067.

[19] Cukierman, A. (1989). Why does the Fed smooth interest rates? In *Monetary Policy on the 75th Anniversary of the Federal Reserve System*, Belgonia M (ed.). Kluwer Academica Press: Boston, MA.

[20] De Long, B. (1997). America's Only Peacetime Inflation: The 1970s. In Romer, C. and Romer, D. Eds., Reducing Inflation, University of Chicago Press, CHicago.

[21] Dennis, R. (2006). The Policy Preferences of the US Federal Reserve. *Journal of Applied Econometrics* 21, 55-77.

[22] Duffy, J. and Xiao, W. (2011). Investment and monetary policy: learning and determinacy of equilibrium. *Journal of Money, Credit and Banking*, 43(5), 959-992.

[23] Erceg, C., Henderson D., and Levin A., (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46, 281-313.

[24] Evans, G., and Honkapohja, S. (1999). Learning dynamics. Handbook of Macroeconomics, 1(A), 449-542.

[25] Evans, G., and Honkapohja, S. (2001). Learning and Expectations in Macroeconomics Monetary. Princeton University Press, Princeton, NJ..

[26] Evans, G., and Honkapohja, S. (2003). Adaptive Learning and Monetary Policy Design. *Journal of Money, Credit and Banking* 35(6), 1045-1073.

[27] Evans, G., and Honkapohja, S. (2009). Expectations, learning and monetary policy: an overview of recent research. In Central Banking Analysis, and Economic Policies Book Series, in Klaus Schmidt-Hebbel, Carl E. Walsh, and Norman Loayza (Series Editors). Monetary Policy under Uncertainty and Learning, Ed. 1, Vol. 13, Ch. 2, pp. 27-76.

[28] Evans, G., and McGough, B. (2005a). Monetary policy, indeterminacy and learning. *Journal of Economic Dynamics and Control* 29, 1809-1840.

[29] Evans, G., and McGough, B. (2005b). Stable sunspot solutions in models with predetermined variables. *Journal of Economic Dynamics and Control* 29, 601 - 625.

[30] Evans, G. and McGough, B. (2005c). Monetary policy and stable indeterminacy.*Economics Letters* 87, 1-7.

[31] Evans, G., and McGough, B. (2007). Optimal constrained interest-rate rules. *Journal of Money, Credit, and Banking* 39(6), 1335-1356.

[32] Flaschel, P., Franke, R., and Proaño, C. (2008). On equilibrium determinacy in New Keynesian models with staggered wage and price setting. *B.E. Journal of Macroeconomics*, 8(1).

[33] Franke, R., and Flaschel, P. (2008). A Proof of Determinacy in the New-Keynesian Sticky Wages and Prices Model. Economics Working Papers 2008,14, Christian-Albrechts-University of Kiel, Department of Economics.

[34] Galí, J. (2008). Monetary policy, inflation and the business cycle: An introduction to the New Keynesian Framework. Princeton University Press.

[35] Giannoni, M., and Woodford, M. (2003). Optimal interest-rate rules: I. General theory. NBER Working Papers 9419.

[36] Huang, K., Meng, Q., and Xue, J. (2009). Is forward-looking inflation targeting destabilizing? The role of policy's response to current output under endogenous investment. *Journal of Economic Dynamics and Control*, 33(2), 409-430.

[37] Kurozumi, T., and Van Zandweghe, W. (2008) Investment, interest rate policy, and equilibrium stability. *Journal of Economic Dynamics and Control*, 32(5), 1489-516.

[38] Levin, A., Onatski, A., Williams, J., and Williams N. (2006). Monetary policy under uncertainty in micro-founded macroeconometric models. in NBER Macroeconomics Annual 2005, Volume 20, MIT Press.

[39] Llosa, L., and Tuesta V. (2008). Determinacy and learnability of monetary policy rules in small open economies. *Journal of Money, Credit and Banking*, 40(5), 1033-1063.

[40] Mankiw, N.G., and Reis, R. (2003). What measure of inflation should a central bank target? *Journal of the European Economic Association*, 1(5): 1058-1086.

[41] Marzo, M. (2009). Wage or price based inflation? Alternative targets in optimal monetary policy rules. *Journal of Economic Dynamics and Control*, 33, 1296-1313.

[42] McGough, B., Rudebusch, G, and Williams, J. (2005). Using a long-term interest rate as the monetary policy Instrument. *Journal of Monetary Economics* 52(5): 855-879.

[43] Pfajfar, D., and Santoro, E. (2012) Credit market distortions, asset prices, and monetary policy. Discussion Paper 2012-010, Tilburg University, Center for Economic Research.

[44] Powell, M.J.D. (1970) A Fortran subroutine for solving systems of nonlinear algebraic equations, Numerical Methods for Nonlinear Algebraic Equations, P. Rabinowitz, ed., Ch.7.

[45] Rotemberg, J., and Woodford, M., (1997). An Optimization-based econometric framework for the evaluation of monetary policy: Expanded version. NBER Technical Working Papers 0233.

[46] Smets, F., and Wouters, R., (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97, 586-606.

[47] Wang, Q. (2006). Learning stability for monetary policy rules in a two-country model. Working Paper 659, University of Cambridge

[48] Woodford, M. (1999). Optimal Monetary Policy Inertia. NBER Working Paper No. 7261.

[49] Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press, Princeton, NJ.

[50] Zanna, L.(2009). PPP exchange rate rules, macroeconomic (in)stability, and learning. *International Economic Review*, 50(4), 1103-1128.

Appendix A. Contemporaneous Data in the Policy Rule

Appendix A.1. ***Matrix for Determinacy***

The model can be simplified by substituting the policy rule (5) into (4) and writing the system involving the endogenous variables x_t, π_t, π_t^w, and w_{t-1} given by Eqs. (1), (2), (3), and (4) in the form of Eq. (8), reproduced here for convenience,

$$y_t = \alpha + BE_t y_{t+1} + \kappa e_t \tag{A.1}$$

where y_t=[x_t, π_t, π_t^w, w_{t-1}]$\prime$, $\alpha = w_t^n$, e_t=[${r_t}^n$, u_t]$\prime$, and matrix B is defined as follows

$$B = g.\begin{pmatrix} 1 & \sigma-\beta\sigma\psi_\pi & -\beta\sigma\psi_{\pi^w} & -\xi_p\sigma\psi_\pi+\xi_w\sigma\psi_{\pi^w} \\ \kappa_p & \beta\left(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x+\kappa_w\sigma\psi_{\pi^w}\right)+\kappa_p\left(\sigma-\beta\sigma\psi_\pi\right) & -\beta\kappa_p\sigma\psi_{\pi^w} & \sigma\kappa_p\xi_w\psi_{\pi^w}+\xi_p\left(1+\sigma\psi_x+\kappa_w\sigma\psi_{\pi^w}\right) \\ \kappa_w & \kappa_w\left(\sigma-\beta\sigma\psi_\pi\right) & \beta\left(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x\right) & -\kappa_w\xi_p\sigma\psi_\pi+\xi_w\left(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x\right) \\ \kappa_p-\kappa_w & \beta+\sigma\kappa_p+\beta\sigma\psi_x+\kappa_w\left(-\sigma+\beta\left(\sigma\psi_\pi+\sigma\psi_{\pi^w}\right)\right) & -\beta\left(1+\sigma\psi_x+\kappa_p\left(\sigma\psi_\pi+\sigma\psi_{\pi^w}\right)\right) & \left(1+\xi_w\right)\left(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x\right)+\left(\kappa_w+\kappa_p\xi_w\right)\sigma\psi_{\pi^w}+\xi_p\left(1+\sigma\psi_x+\kappa_w\left(\sigma\psi_\pi+\sigma\psi_{\pi^w}\right)\right) \end{pmatrix},$$

where $g = \frac{1}{1+\kappa_p\sigma\psi_\pi+\sigma\psi_x+\kappa_w\psi_{\pi w}}$.

The characteristic polynomial of the inverse of B is $p(\lambda) = \lambda^4+a_3\lambda^3+a_2\lambda^2+a_1\lambda+a_0$ where

$a_0 = \frac{\sigma(\kappa_p\psi_\pi+\kappa_w\psi_{\pi w}+\psi_x)+1}{\beta^2}$,

$a_1 = -\frac{\xi_p+\xi_w+\beta(\kappa_p\sigma\psi_\pi+\kappa_w\sigma\psi_{\pi w}+2\sigma\psi_x+2)+\sigma(\psi_x(\xi_p+\xi_w+1)+\xi_w\kappa_p(\psi_\pi+\psi_{\pi w})+\kappa_w(\xi_p(\psi_\pi+\psi_{\pi w})+\psi_{\pi w}))+\kappa_p\sigma(\psi_\pi+1)+2}{\beta^2}$,

$a_2 = \frac{\beta^2(\sigma\psi_x+1)+\xi_p+\xi_w+\beta(\sigma\psi_x(\xi_p+\xi_w+2)+\xi_p+\xi_w+\kappa_p\sigma\psi_\pi+\kappa_p\sigma+\kappa_w\sigma\psi_{\pi w}+4)+\sigma(\xi_w\kappa_p+\kappa_p+\xi_p\kappa_w)+1}{\beta^2}$

and

$a_3 = -\frac{\beta\sigma\psi_x+2\beta+\xi_p+\xi_w+\kappa_p\sigma+2}{\beta}$.

Following the analysis in Carlstrom et al. (2006) and Carlstrom and Fuerst (2007), the modified Taylor principle for prices and wages Eq. (22) is given by $p(1) < 0$ where $p(1) = \frac{(\beta-1)\sigma\psi_x(\xi_p+\xi_w)-\sigma(\psi_\pi+\psi_{\pi w}-1)(\xi_w\kappa_p+\xi_p\kappa_w)}{\sigma(\kappa_p\psi_\pi+\kappa_w\psi_{\pi w}+\psi_x)+1}$.

Appendix A.2. ***Matrices for Learning.***

The model is written as

$$\widehat{y}_t = \alpha+\widehat{B}E_t\widehat{y}_{t+1}+\widehat{\delta}\widehat{y}_{t-1}+\widehat{\kappa}\ e_t, \tag{A.2}$$

where $\widehat{y}_t$=$[x_t,\ \pi_t,\ \pi_t^w,\ w_t]\prime$, $\alpha = w_t^n$, e_t=$[{r_t}^n,\ u_t]\prime$, and matrices $\widehat{B}$, $\widehat{\delta}$, and $\widehat{\kappa}$ are defined as

$$\widehat{B} = f.\begin{pmatrix} 1+\xi_p+\xi_w & \sigma+\sigma\xi_p-\beta\sigma\psi_\pi+\xi_w\left(\sigma-\beta\left(\sigma\psi_\pi+\psi_{\pi w}\right)\right) & -\beta\left(\psi_{\pi w}+\xi_p\left(\sigma\psi_\pi+\psi_{\pi w}\right)\right) & 0 \\ \kappa_w\xi_p+\kappa_p\left(1+\xi_w\right) & \sigma\kappa_p\left(1+\xi_w\right)+\beta\left(1+\xi_w\right)\left(1+\sigma\psi_x\right)+\kappa_w\left(\sigma\xi_p+\beta\psi_{\pi w}\right) & \beta\left(\xi_p\left(1+\sigma\psi_x\right)-\kappa_p\psi_{\pi w}\right) & 0 \\ \kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w & \kappa_w\left(\sigma+\sigma\xi_p-\beta\sigma\psi_\pi\right)+\xi_w\left(\beta+\sigma\kappa_p+\beta\sigma\psi_x\right) & \beta\left(1+\xi_p+\kappa_p\sigma\psi_\pi+\sigma\left(1+\xi_p\right)\psi_x\right) & 0 \\ -\kappa_p+\kappa_w & -\beta+\sigma\kappa_p+\beta\sigma\psi_x+\kappa_w\left(-\sigma+\beta\left(\sigma\psi_\pi+\psi_{\pi w}\right)\right) & \beta\left(1+\sigma\psi_x+\kappa_p\left(\sigma\psi_\pi+\psi_{\pi w}\right)\right) & 0 \end{pmatrix},$$

$$\widehat{\delta} = \begin{pmatrix} 0 & 0 & 0 & \frac{-2\xi_p\sigma\psi_\pi+2\xi_w\psi_{\pi w}}{\xi_w(1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)} \\ 0 & 0 & 0 & \frac{2(\kappa_p\xi_w\psi_{\pi w}+\xi_p(1+\sigma\psi_x+\kappa_w\psi_{\pi w}))}{\xi_w(1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)} \\ 0 & 0 & 0 & -\frac{2(\kappa_w\xi_p\sigma\psi_\pi+\xi_w(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x))}{\xi_w(1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)} \\ 0 & 0 & 0 & \frac{-(-1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)+(\kappa_w-\kappa_p\xi_w)\psi_{\pi w}-\xi_p(1+\sigma\psi_x+\kappa_w(\sigma\psi_\pi+\psi_{\pi w}))}{\xi_w(1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)} \end{pmatrix}, \text{ and}$$

$$\widehat{\kappa} = f.\begin{pmatrix} \sigma\left(1+\xi_p+\xi_w\right) & -\kappa_p\left(\left(1+\xi_w\right)\sigma\psi_\pi+\xi_w\psi_{\pi^w}\right)+\kappa_w\left(\psi_{\pi^w}+\xi_p\left(\sigma\psi_\pi+\psi_{\pi^w}\right)\right) \\ \sigma\left(\kappa_w\xi_p+\kappa_p\left(1+\xi_w\right)\right) & \left(\kappa_w\xi_p+\kappa_p\left(1+\xi_w\right)\right)\left(1+\sigma\psi_x\right) \\ \sigma\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right) & \left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)\left(1+\sigma\psi_x\right) \\ \sigma\left(-\kappa_p+\kappa_w\right) & -\left(\kappa_p-\kappa_w\right)\left(1+\sigma\psi_x\right) \end{pmatrix},$$

where $f = \frac{1}{(1+\xi_w)\left(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x\right)+\left(\kappa_w+\kappa_p\xi_w\right)\psi_{\pi w}+\xi_p(1+\sigma\psi_x+\kappa_w(\sigma\psi_\pi+\psi_{\pi w}))}$.

The E-stability analysis follows Section 2.4.

Appendix B. Forward Expectations in the Policy Rule

Appendix B.1. ***Matrix for Determinacy***

The model with forward-looking expectations is composed of Eqs. (1)-(4) and (6). The system can be reduced to four equations when I substitute (6) into (4). The system can be rewritten involving the endogenous variables x_t, π_t, π_t^w, and w_{t-1} given by Eqs. (1), (2), (3), and (4) in the form of Eq. (A.1). Matrix B is defined here as

$$B = \begin{pmatrix} 1-\sigma\psi_x & \sigma-\sigma\psi_\pi & -\sigma\psi_{\pi^w} & 0 \\ \kappa_p\left(1-\sigma\psi_x\right) & \beta+\kappa_p\left(\sigma-\sigma\psi_\pi\right) & -\kappa_p\sigma\psi_{\pi^w} & \xi_p \\ \kappa_w\left(1-\sigma\psi_x\right) & \kappa_w\left(\sigma-\sigma\psi_\pi\right) & \beta-\kappa_w\sigma\psi_{\pi^w} & -\xi_w \\ \left(\kappa_p-\kappa_w\right)\left(1-\sigma\psi_x\right) & \beta+\left(\kappa_p-\kappa_w\right)\left(\sigma-\sigma\psi_\pi\right) & -\beta-\left(\kappa_p-\kappa_w\right)\sigma\psi_{\pi^w} & 1+\xi_p+\xi_w \end{pmatrix}.$$

The characteristic polynomial of B is $p(\lambda)=\lambda^4+a_3\lambda^3+a_2\lambda^2+a_1\lambda+a_0$ where
$a_0=\beta^2(1-\sigma\psi_x)$,
$a_1=\beta\sigma\psi_x(\beta+\xi_p+\xi_w+2)-\beta(2\beta+\xi_p+\xi_w+\sigma(\kappa_p(-\psi_\pi)+\kappa_p-\kappa_w\psi_{\pi^w})+2)$,
$a_2=\beta^2+\xi_p+\xi_w+\beta\xi_p+\xi_w-\sigma(\kappa_p(\psi_\pi-1)+\kappa_w\psi_{\pi^w}+2\psi_x)+4)-\sigma(\psi_x(\xi_p+\xi_w+1)+\kappa_p(\xi_w(\psi_\pi+\psi_{\pi^w}-1)+\psi_\pi)+\kappa_w(\xi_p(\psi_\pi+\psi_{\pi^w}-1)+\psi_{\pi^w}))+\kappa_p\sigma+1$, and
$a_3=-2\beta-\xi_p-\xi_w+\sigma(\kappa_p(\psi_\pi-1)+\kappa_w\psi_{\pi^w}+\psi_x)-2$.

With $p(1)=(\beta-1)\sigma\psi_x(\xi_p+\xi_w)-\sigma(\psi_\pi+\psi_{\pi^w}-1)(\xi_w\kappa_p+\xi_p\kappa_w)$ and $p(-1)=\beta^2(4-2\sigma\psi_x)-\beta\sigma\psi_x(\xi_p+\xi_w+4)+2(\xi_p+\xi_w+\kappa_p\sigma+2)+2\beta(\xi_p+\xi_w+\sigma(\kappa_p(-\psi_\pi)+\kappa_p-\kappa_w\psi_{\pi^w})+4)-\sigma(\psi_x(\xi_p+\xi_w+2)+\kappa_p(\xi_w(\psi_\pi+\psi_{\pi^w}-1)+2\psi_\pi)+\xi_p\kappa_w(\psi_\pi-1)+(\xi_p+2)\kappa_w\psi_{\pi^w})$, where $p(1)<0$ implies (22) and $p(-1)>0$ by (24).

Appendix B.2. ***Matrices for Learning***

The model is written as in Eq. (A.2), where $\widehat{y}_t=[x_t,\ \pi_t,\ \pi_t^w,\ w_t]\prime$, $\alpha=w_t^n$, $e_t=[{r_t}^n,\ u_t]\prime$, and matrices $\widehat{B}$, $\widehat{\delta}$, and $\widehat{\kappa}$ are

$$\widehat{B} = \begin{pmatrix} 1-\sigma\psi_x & \sigma-\sigma\psi_\pi & -\sigma\psi_{\pi^w} & 0 \\ \frac{\left(\kappa_w\xi_p+\kappa_p(1+\xi_w)\right)(1-\sigma\psi_x)}{1+\xi_p+\xi_w} & \frac{\beta(1+\xi_w)+\kappa_w\xi_p(\sigma-\sigma\psi_\pi)+\kappa_p(1+\xi_w)(\sigma-\sigma\psi_\pi)}{1+\xi_p+\xi_w} & \frac{-\kappa_p(1+\xi_w)\sigma\psi_\pi w+\xi_p(\beta-\kappa_w\sigma\psi_\pi w)}{1+\xi_p+\xi_w} & 0 \\ \frac{\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)(1-\sigma\psi_x)}{1+\xi_p+\xi_w} & \frac{\xi_w\left(\beta+\kappa_p(\sigma-\sigma\psi_\pi)\right)+\kappa_w\left(1+\xi_p\right)(\sigma-\sigma\psi_\pi)}{1+\xi_p+\xi_w} & \frac{\beta\left(1+\xi_p\right)-\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)\sigma\psi_\pi w}{1+\xi_p+\xi_w} & 0 \\ \frac{\left(\kappa_p-\kappa_w\right)(-1+\sigma\psi_x)}{1+\xi_p+\xi_w} & \frac{-\beta+\kappa_w(\sigma-\sigma\psi_\pi)+\kappa_p(-\sigma+\sigma\psi_\pi)}{1+\xi_p+\xi_w} & \frac{\beta+\kappa_p\sigma\psi_\pi w-\kappa_w\sigma\psi_\pi w}{1+\xi_p+\xi_w} & 0 \end{pmatrix},$$

$$\widehat{\delta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\xi_p}{1+\xi_p+\xi_w} \\ 0 & 0 & 0 & -\frac{\xi_w}{1+\xi_p+\xi_w} \\ 0 & 0 & 0 & \frac{1}{1+\xi_p+\xi_w} \end{pmatrix}, and\ \widehat{\kappa} = \begin{pmatrix} \sigma & 0 \\ \frac{\sigma\left(\kappa_w\xi_p+\kappa_p(1+\xi_w)\right)}{1+\xi_p+\xi_w} & \frac{\kappa_w\xi_p+\kappa_p(1+\xi_w)}{1+\xi_p+\xi_w} \\ \frac{\sigma\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)}{1+\xi_p+\xi_w} & \frac{\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w}{1+\xi_p+\xi_w} \\ \frac{\sigma\left(-\kappa_p+\kappa_w\right)}{1+\xi_p+\xi_w} & \frac{-\kappa_p+\kappa_w}{1+\xi_p+\xi_w} \end{pmatrix}.$$

Determinacy analysis was completed according to Section 2.3 and E-stability analysis conforms to Section 2.4

Appendix C. Lagged Data in the Policy Rule

Appendix C.1. ***Matrices for Determinacy***

The model encompasses Eqs. (1)-(4) and (7), where Eq. (7) has been moved one period forward. The system is rewritten as

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \sigma \\ -\kappa_p & 1 & 0 & 0 & 0 \\ -\kappa_w & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ -\psi_x & -\psi_\pi & -\psi_{\pi^w} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \\ {\pi_t}^w \\ w_{t-1} \\ i_t \end{pmatrix} = \begin{pmatrix} 1 & \sigma & 0 & 0 & 0 \\ 0 & \beta & 0 & \xi_p & 0 \\ 0 & 0 & \beta & -\xi_w & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} Ex_{t+1} \\ E\pi_{t+1} \\ E\pi_{t+1}^w \\ w_t \\ i_{t+1} \end{pmatrix} \tag{C.1}$$

$$+ \begin{pmatrix} 0 \\ -\xi_p \\ \xi_w \\ 0 \\ 0 \end{pmatrix} {w_t}^n + \begin{pmatrix} \sigma & 0 \\ 0 & \kappa_p \\ 0 & \kappa_w \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} {r_t}^n \\ u_t \end{pmatrix}.$$

The matrix of expectational variables pre-multiplies the inverse of the first (left hand) matrix from Eq. (C.1) resulting in

$$B = h.\begin{pmatrix} 0 & -\beta\psi_\pi & -\beta\psi_{\pi^w} & -\xi_p\psi_\pi + \xi_w\psi_{\pi^w} & 1 \\ 0 & \beta - \frac{\beta\kappa_p\psi_\pi}{\kappa_p\psi_\pi+\psi_x+\kappa_w\psi_{\pi^w}} & -\beta\kappa_p\psi_{\pi^w} & \xi_p\psi_x + \left(\kappa_w\xi_p + \kappa_p\xi_w\right)\psi_{\pi^w} & \kappa_p \\ 0 & -\beta\kappa_w\psi_\pi & \beta\left(\kappa_p\psi_\pi + \psi_x\right) & -\kappa_w\xi_p\psi_\pi + \xi_w\left(\kappa_p\psi_\pi + \psi_x\right) & \kappa_w \\ 0 & \beta\left(\psi_x + \kappa_w\left(\psi_\pi + \psi_{\pi^w}\right)\right) & -\beta\left(\psi_x + \kappa_p\left(\psi_\pi + \psi_{\pi^w}\right)\right) & \left(1+\xi_p+\xi_w\right)\psi_x + \kappa_p\left(\left(1+\xi_w\right)\psi_\pi + \xi_w\psi_{\pi^w}\right) + \kappa_w\left(\psi_{\pi^w} + \xi_p\left(\psi_\pi + \psi_{\pi^w}\right)\right) & \kappa_p - \kappa_w \\ \frac{\kappa_p\psi_\pi+\psi_x+\kappa_w\psi_{\pi^w}}{\sigma} & \kappa_p\psi_\pi + \psi_x + \kappa_w\psi_{\pi^w} + \frac{\beta\psi_\pi}{\sigma} & \frac{\beta\psi_{\pi^w}}{\sigma} & \frac{\xi_p\psi_\pi - \xi_w\psi_{\pi^w}}{\sigma} & -\frac{1}{\sigma} \end{pmatrix},$$

where $h = \frac{1}{\kappa_p\psi_\pi+\psi_x+\kappa_w\psi_{\pi^w}}$.

Appendix C.2. ***Matrices for Learning***

The model was reshaped by substituting equation (7) into equation (4) and is written in terms of the endogenous variables x_t, π_t, π_t^w, and w_t as in Eq. (A.2). Matrices $\widehat{B}$, $\widehat{\delta}$, and $\widehat{\kappa}$ are

$$\widehat{B} = \begin{pmatrix} 1 & \sigma & 0 & 0 \\ \frac{\kappa_w\xi_p+\kappa_p(1+\xi_w)}{1+\xi_p+\xi_w} & \frac{\beta+\sigma\kappa_w\xi_p+\beta\xi_w+\sigma\kappa_p(1+\xi_w)}{1+\xi_p+\xi_w} & \frac{\beta\xi_p}{1+\xi_p+\xi_w} & 0 \\ \frac{\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w}{1+\xi_p+\xi_w} & \frac{\sigma\kappa_w\left(1+\xi_p\right)+\left(\beta+\sigma\kappa_p\right)\xi_w}{1+\xi_p+\xi_w} & \frac{\beta\left(1+\xi_p\right)}{1+\xi_p+\xi_w} & 0 \\ \frac{-\kappa_p+\kappa_w}{1+\xi_p+\xi_w} & -\frac{\beta+\sigma\kappa_p-\sigma\kappa_w}{1+\xi_p+\xi_w} & \frac{\beta}{1+\xi_p+\xi_w} & 0 \end{pmatrix},$$

$$\widehat{\delta} = \begin{pmatrix} -\sigma\psi_x & -\sigma\psi_\pi & -\sigma\psi_{\pi^w} & 0 \\ -\frac{\left(\kappa_w\xi_p+\kappa_p(1+\xi_w)\right)\sigma\psi_x}{1+\xi_p+\xi_w} & -\frac{\left(\kappa_w\xi_p+\kappa_p(1+\xi_w)\right)\sigma\psi_\pi}{1+\xi_p+\xi_w} & -\frac{\left(\kappa_w\xi_p+\kappa_p(1+\xi_w)\right)\sigma\psi_{\pi^w}}{1+\xi_p+\xi_w} & \frac{\xi_p}{1+\xi_p+\xi_w} \\ -\frac{\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)\sigma\psi_x}{1+\xi_p+\xi_w} & -\frac{\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)\sigma\psi_\pi}{1+\xi_p+\xi_w} & -\frac{\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)\sigma\psi_{\pi^w}}{1+\xi_p+\xi_w} & -\frac{\xi_w}{1+\xi_p+\xi_w} \\ \frac{\left(\kappa_p-\kappa_w\right)\sigma\psi_x}{1+\xi_p+\xi_w} & \frac{\left(\kappa_p-\kappa_w\right)\sigma\psi_\pi}{1+\xi_p+\xi_w} & \frac{\left(\kappa_p-\kappa_w\right)\sigma\psi_{\pi^w}}{1+\xi_p+\xi_w} & \frac{1}{1+\xi_p+\xi_w} \end{pmatrix},$$

and $$\widehat{\kappa} = \begin{pmatrix} \sigma & 0 \\ \frac{\sigma\left(\kappa_w\xi_p+\kappa_p(1+\xi_w)\right)}{1+\xi_p+\xi_w} & \frac{\kappa_w\xi_p+\kappa_p(1+\xi_w)}{1+\xi_p+\xi_w} \\ \frac{\sigma\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)}{1+\xi_p+\xi_w} & \frac{\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w}{1+\xi_p+\xi_w} \\ \frac{\sigma\left(-\kappa_p+\kappa_w\right)}{1+\xi_p+\xi_w} & \frac{-\kappa_p+\kappa_w}{1+\xi_p+\xi_w} \end{pmatrix}.$$

Determinacy and E-stability analyses were performed following Sections 2.3 and 2.4, respectively.